

Altruistic behavior pays, or the importance of fluctuations in evolutionary game theory

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Abstract. Human behavior is one of the main problems for evolution, as it is often the case that human actions are disadvantageous for the self and advantageous for other people. Behind this puzzle are our beliefs about rational behavior, based on game theory. Here we show that by going beyond the standard game-theoretical conventions, apparently altruistic behavior can be understood as self-interested. We discuss in detail an example related to the so called Ultimatum game and illustrate the appearance of altruistic behavior induced by fluctuations. In addition, we claim that in general settings, fluctuations play a very relevant role, and we support this claim by considering a completely different example, namely the Stag-Hunt game.

INTRODUCTION

Human altruistic behavior is a long-standing problem in evolutionary theory, as first realized by Darwin himself:

He who was ready to sacrifice his life (...) rather than betray his comrades, would often leave no offspring to inherit his noble nature... Therefore, it seems scarcely possible (...) that the number of men gifted with such virtues (...) would be increased by natural selection, that is, by the survival of the fittest. [1]

At the crux of the problem lies the fact that Darwin developed his theory assuming that natural selection acts exclusively on individuals. On this ground, he could not possibly understand altruistic behavior in humans, i.e., acts that decrease the actor's fitness while increasing that of others. Reluctantly, he had to call for selection at group level:

A man who was not impelled by any deep, instinctive feeling, to sacrifice his life for the good of others, yet was roused to such actions by a sense of glory, would by his example excite the same wish for glory in other men, and would strengthen by exercise the noble feeling of admiration. He might thus do far more good to his tribe than by begetting offsprings with a tendency to inherit his own high character. [1]

In fact, human behavior is unique in nature. Indeed, altruism or cooperative behavior exists in other species, but it can be understood in terms of genetic relatedness (kin selection, introduced by Hamilton [2]) or of repeated interactions (as proposed by

Trivers [3]). However, human cooperation extends to genetically unrelated individuals and to large groups, characteristics that cannot be understood within those schemes. Subsequently, a number of theories based on group and/or cultural evolution have been put forward in order to explain altruism (see [4] for a review).

THE ULTIMATUM GAME

In order to address quantitatively the issues above, behavioral researchers use evolutionary game theory [5, 6] to design experiments that try to find the influence of different factors. In this paper, we analyze this problem in the context of a specific set of such experiments, related to the so called Ultimatum game [7, 8]. In the Ultimatum game, under conditions of anonymity, two players are shown a sum of money, say 100 €. One of the players, the “proposer”, is instructed to offer any amount, from 1 € to 100 €, to the other, the “responder”. The proposer can make only one offer, which the responder can accept or reject. If the offer is accepted, the money is shared accordingly; if rejected, both players receive nothing. Since the game is played only once (no repeated interactions) and anonymously (no reputation gain; for more on explanations of altruism relying on reputation see [9]), a self-interested responder will accept any amount of money offered. Therefore, self-interested proposers will offer the minimum possible amount, 1 €, which will be accepted. Notwithstanding, in actual Ultimatum game experiments with human subjects, average offers do not even approximate the self-interested prediction. Generally speaking, proposers offer respondents very substantial amounts (50 % being a typical modal offer) and respondents frequently reject offers below 30 % [10]. Most of the experiments have been carried out with university students in western countries, showing a large degree of individual variability but a striking uniformity between groups in average behavior. A large study in 15 small-scale societies [8] found that, in all cases, respondents or proposers behave in a reciprocal manner. Furthermore, the behavioral variability across groups was much larger than previously observed: while mean offers in the case of university students are in the range 43%-48%, in the cross-cultural study they ranged from 26% to 58%.

The fact that indirect reciprocity is excluded by the anonymity condition and that interactions are one-shot (i.e., repeated interaction does not apply) allows one to interpret rejections in terms of the so-called strong reciprocity [11, 12]. This amounts to considering that these behaviors are truly altruistic, i.e., that they are costly for the individual performing them in so far as they do not result in direct or indirect benefit. As a consequence, we return to our evolutionary puzzle: The negative effects of altruistic acts must decrease the altruist’s fitness as compared to that of the recipients of the benefit, ultimately leading to the extinction of altruists. Indeed, standard evolutionary game theory arguments applied to the Ultimatum game lead to the expectation that in a mixed population, punishers (individuals who reject low offers) have less chance to survive than rational players (individuals who accept any offer) and eventually disappear. In the remainder of the paper, we will show that this conclusion depends on the dynamics, and that different dynamics leads to the survival of punishers through fluctuations.

THE MODEL

We consider a population of N players (agents) of the Ultimatum game with a fixed sum of money M per game. Random pairs of players are chosen, of which one is the proposer and another one is the respondent. In its simplest version, we will assume that players are capable of other-regarding behavior (empathy); consequently, in order to optimize their gain, proposers offer the minimum amount of money that they would accept. Every agent has her own, fixed acceptance threshold, $1 \leq t_i \leq M$ (t_i are always integer numbers for simplicity). Agents have only one strategy: respondents reject any offer smaller than their own acceptance threshold, and accept offers otherwise. Money shared as a consequence of accepted offers accumulates to the capital of each of the involved players. As our main aim is to study selection acting on modified descendants, hereafter we interpret this capital as 'fitness' (here used in a loose, Darwinian sense, not in the more restrictive one of reproductive rate). After s games, the agent with the overall minimum fitness is removed (randomly picked if there are several) and a new agent is introduced by duplicating that with the maximum fitness, i.e., with the same threshold and the same fitness (again randomly picked if there are several). Mutation is introduced in the duplication process by allowing changes of ± 1 in the acceptance threshold of the newly generated player with probability $1/3$ each. Agents have no memory (i.e., interactions are one-shot) and no information about other agents (i.e., no reputation gains are possible). We stress that the model is dramatically simplified; however, we have studied more complicated versions (including separate acceptance and offer thresholds) and the results are similar to the ones we discuss below. Another factor we have considered is smaller mutation rates, again without qualitative changes in the result. Therefore, for the sake of brevity we concentrate here on the simple model summarized above and refer the reader to [13] for a more detailed analysis including those other versions.

RESULTS

Figure 1 shows the typical outcome of simulations of our model. As we can see, the mean acceptance threshold rapidly evolves towards values around 40%, while the whole distribution of thresholds converges to a peaked function, with the range of acceptance thresholds for the agents covering about a 10% of the available ones. These are values compatible with the experimental results discussed above. The mean acceptance threshold fluctuates during the length of the simulation, never reaching a stationary value for the durations we have explored. The width of the peak fluctuates as well, but in a much smaller scale than the position. The fluctuations are larger for smaller values of s , and when s becomes of the order of N or larger, the evolution of the mean acceptance threshold is very smooth. This is a crucial point and will be discussed in more detail below. Importantly, the typical evolution we are describing does not depend on the initial condition. In particular, a population consisting solely of self-interested agents, i.e., all initial thresholds are set to $t_i = 1$, evolves in the same fashion. Indeed, the distributions shown in the left panel of Figure 1 have been obtained with such an initial condition, and it can

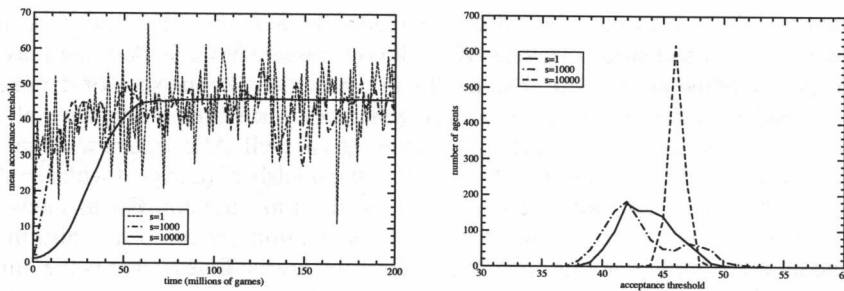


FIGURE 1. Left: mean acceptance threshold as a function of simulation time. Initial condition is that all agents have $t_i = 1$. Right: Acceptance threshold distribution after 10^8 games. Initial condition is that all agents have uniformly distributed, random t_i . s is as indicated from the plot.

be clearly observed that self-interested agents disappear in the early stages of the evolution. The number of players and the value M of the capital at stake in every game are not important either, and increasing M only leads to a higher resolution of the threshold distribution function.

DISCUSSION

As we mentioned in the preceding section, we have observed that taking very large values for s or, strictly speaking, considering the limit $s/N \rightarrow \infty$, does lead to different results. In this respect, let us recall previous studies of the Ultimatum game by Page and Nowak [14, 15]. The model introduced in those works has a dynamics completely different from ours: following standard evolutionary game theory, every player plays every other one in both roles (proponent and respondent), and afterwards players reproduce with probability proportional to their payoff (which is fitness in the reproductive sense). Simulations and adaptive dynamics equations show then that the population ends up composed by players with fair (50%) thresholds. This is different from our observations, in which we hardly ever reach an equilibrium (only for large s) and even then equilibria set up at values different from the fair share. The reason for this difference is that the Page-Nowak model dynamics describes the $s/N \rightarrow \infty$ limit of our model, in which between death-reproduction events the time average gain all players obtain is the mean payoff with high accuracy. We thus see that our model is more general because it has one free parameter, s , that allows selecting different regimes whereas the Page-Nowak dynamics is only one limiting case. Those different regimes are what we have described as fluctuation dominated (when s/N is finite and not too large) and the regime analyzed by Page and Nowak (when $s/N \rightarrow \infty$). This amounts to saying that by varying s we can study regimes far from the standard evolutionary game theory limit. As a result, we find a variability of outcomes for the acceptance threshold consistent with the observations in real human societies [8, 10].

In fact, fluctuations due to the finite number of games are at the heart of our results. Among the results summarized above, the evolution of a population entirely formed by

self-interested players into a diversified population with a large majority of altruists is the most relevant and surprising one. We will now argue that the underlying reason for this is precisely the presence of fluctuations in our model. For the sake of definiteness, let us consider the case $s = 1$ (agent replacement takes place after every game) although the discussion applies to larger (but finite) values of s as well. After one or more games, a mutation event will take place and a “weak altruistic punisher” (an agent with $t_i = 2$) will appear in the population, with a fitness inherited from its ancestor. For this new agent to be removed at the next iteration so that the population reverts to its uniform state, our model rules imply that this agent has to have the lowest fitness, that is the only one with that value of fitness, *and also* that it does not play as a proposer in the next game (if playing as a responder the agent will earn nothing because of her threshold). In any other event this altruistic punisher will survive at least one cycle, in which an additional one can appear by mutation. It is thus clear that fluctuations indeed help altruists to take over: As soon as a few altruists are present in the population, it is easy to see analytically that they will survive and proliferate even in the limit $s/N \rightarrow \infty$.

THE STAG-HUNT GAME

This far, we have shown that considering that players play a finite number of games between death-birth events in the Ultimatum game leads to results unexpected from standard evolutionary game theory arguments. Hence, the question arises as to whether this is a consequence of the many strategies available in the Ultimatum game (as many as possible values for t_i , 100 with our choice for the parameters) or, on the contrary, it is a general phenomenon. To show that the latter is the case, we have considered a completely different, much simpler kind of game: the so-called Stag-Hunt game [5, 6, 8]. In this game, two hunters cooperate in hunting for stag, which is the most profitable option; however, hunting a stag is impossible unless both work together, and they have the option of hunting for rabbit, less profitable, but with sure earnings. This is reflected in the following payoff matrix (C stands for cooperation in hunting stag, D stands for defection and hunting rabbit alone):

	C	D
C	6	0
D	5	1

This game belongs in the class of coordination games: In the language of game theory, it has two Nash equilibria, (C,C) and (D,D), and the players would like to coordinate in choosing the first one (so called payoff-dominant). However, the second one is a safer choice because it has the largest guaranteed minimum payoff (so called risk-dominant).

We have been working on the evolutionary dynamics of this game and, specifically, on the equilibrium selection problem [16]. For this example, we have chosen the dynamics given by the Moran process [17], in which after s games an agent is duplicated with probability proportional to the fitness accumulated during the s games, and another one is killed randomly. With such a simple dynamics, it is an elementary exercise to show

that, in the limit $s/N \rightarrow \infty$, the whole population becomes C (resp. D) strategists if the initial density of C strategists is larger (resp. smaller) than 1/2. As Fig. 2 shows, simulation results for finites are largely different from that analytical prediction:

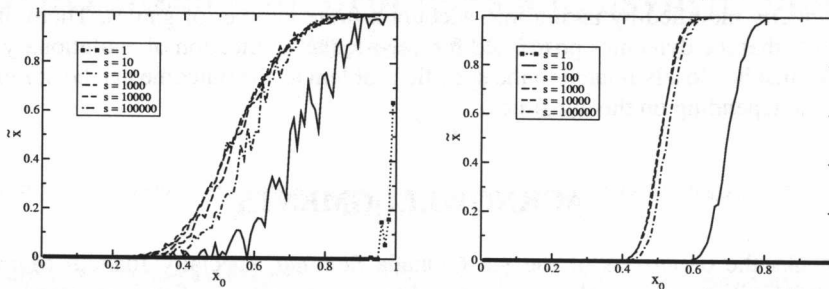


FIGURE 2. Fraction of games that end up with a cooperator-only population vs density of cooperators in the initial state for $N = 100$ (up) and $N = 1000$ (down) agents playing the Stag-Hunt game. Results are obtained from simulations of the Stag-Hunt game with the Moran dynamics, and for every initial density the final density is averaged over 100 games. Values of s are as indicated in the plot.

Indeed, we see that for cooperators to prevail in the final state, an initial density larger than 1/2 is needed. In particular, for $s = 1$, all agents become defectors except for initial densities close to 1 in the case $N = 100$ (left panel), and for all initial densities for $N = 1000$ (right panel) or larger (not shown). The plots also show that larger populations lead to better statistics (meaning that curves are smoother and less noisy; it is evident that \bar{x} has a smaller variance for larger populations), and the trend upon increasing N is that the curves become step functions (as should be for an infinite population). Importantly, the effect, namely that the basin of attraction of the (D,D) equilibrium is enlarged for finite s , persists even in the infinite population limit. In addition, it is also robust upon changes in the dynamics: we have verified that choosing the agent to be eliminated with probability inversely proportional to the agent's fitness leads to qualitatively similar result. We are thus faced with another clear-cut manifestation of the relevance of taking the limit of infinite games before the dynamics occurs or, on the contrary, sticking to a finite number of games. Once again, we stress that the setup is completely different from the Ultimatum game and, as a consequence, we claim that this kind of phenomena is generic and should be observed in many other problems.

CONCLUSIONS

In this paper, we have shown that altruistic-like behavior, specifically, altruistic punishment, may arise by means of exclusive individual selection even in the absence of repeated interactions and reputation gains. Our conclusion is important in so far as it is generally believed that some kind of group selection is needed to understand the observed human behavior. The reason for that is that game theoretical arguments apparently show that altruists are at disadvantage with respect to selfish individual. In this respect, another relevant conclusion of the present work is that perspectives and approaches alternative to standard evolutionary game theory may be needed in order

to understand paradoxical features such as the appearance of altruistic punishment. As additional evidence supporting this claim, we have briefly discussed, in the context of the much simpler problem of the stag-hunt game, that equilibrium selection is indeed dramatically modified by taking into account a finite number of games. Therefore, we conclude that the dynamics postulated for a particular application of evolutionary game theory must be closely related to the specific problem as the outcome can be completely different depending on the dynamics.

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